PHYSICS 415 - Fall Semester 2025 - ODU

Nuclear and Particle Physics - Problem Set 2

Problem 1)

a) A beam of 10^{11} neutrons per second impinges on a target of liquid hydrogen (length: 10 cm, density of liquid hydrogen: 0.0708 g/cm^3). The neutrons scatter elastically and isotropically. The elastic cross section is $d\sigma/d\Omega = 40$ mb/sr (milli-barn per steradian). A detector with area 10 cm^2 is positioned 10 m away from the target. Calculate the rate of neutrons detected in this detector.

b) The CLAS12 detector in Hall B at Jefferson Lab can operate at a ("nuclear") luminosity of at most $L=10^{34}$ /cm^2/s for light nuclei. If you have a 5 cm long carbon-12 (^{12}C) target in Hall B, what is the maximum useful electron beam current that you can shoot at that target?

Problem 2)

a) Assume a light particle, like a slow, **non**-relativistic electron, with momentum k scatters elastically off a very heavy nucleus at some angle θ_e, with final momentum k' (same magnitude as k, but different direction!) Calculate the value of the momentum transferred, q, to that nucleus in terms of the scattering angle and the initial electron energy, if you can ignore the recoil energy of the heavy nucleus.

b) Repeat this calculation for an **ultra**-relativistic electron where you cannot ignore the recoil energy (i.e., the final electron energy E' is less than the incoming beam energy E) – but you may treat the electron mass as infinitely small *(i.e.*, set it to zero). Assume that the incoming electron has four-vector momentum $k^{\mu} = (E,0,0,E)$ (moving along the z-axis, with the first = 0-component of the four vector being the energy), and the scattered electron has 4-vector $k'^{\mu} = (E', E' \sin\theta_e \cos\phi_e, E' \sin\theta_e \sin\phi_e, E' \cos\theta_e)$.
Note that we simply omit all factors of c ! Again, calculate the magnitude of the 3-vector part of the 4-momentum transferred.

c) For the case b), calculate the invariant square of the four-momentum transferred and show it has the magnitude:
$Q^2 := -q^{\mu} q_{\mu} := (k^{\mu} - k'^{\mu})(k_{\mu} - k'_{\mu}) = 4EE' \sin^2(\theta_e/2)$.

(We'll calculate E' later).
Hint: Simplify your math by recalling that $\cos 2\theta = \cos^2\theta - \sin^2\theta$.